
Mixed Precision Domain Decomposition Preconditioners: Perturbation Analysis and Practical Guidelines

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Abstract

This work is concerned with domain decomposition preconditioners, specifically the Schwarz method with GENE0 coarse space, to solve large, sparse symmetric positive definite problems. Through libraries such as HPDDM, these preconditioners have already been efficiently parallelized in their numerical implementations. However, they still require expensive linear algebra operations in each local subdomain. Motivated by the emergence of fast low precision arithmetic in hardware, we aim in this work to speed them up using mixed precision. To do this, we need to identify the sensitivity of these operations to perturbation and propose an actionable criteria for selecting the appropriate precision for each local subdomain. In order to do so, we develop a perturbation theory for our preconditioner to bound the worst-case loss of efficiency of the preconditioner and study the sharpness of this theoretical bound through numerical experiments with the FreeFEM, petsc4py, and HPDDM libraries. Our findings show that the only important parameter is the maximum of the sizes of the local subdomain perturbations, weighted by the condition number of the local subdomain matrix. Our results therefore suggest that preconditioners can be constructed in mixed precision while effectively controlling the loss of efficiency.

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