Two-level asynchronous substructuring domain decomposition solvers

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Abstract

Domain decomposition methods (DDMs) are well established as providing state-of-the-art coarse-grain parallelism, essentially by optimizing data distribution. Domain decomposition was first presented by H. A. Schwarz as a mathematical proof technique for elliptic partial differential equations (PDEs), where the subdomains were of regular shapes and partially overlapping. In the explicit parallel computing context, first DDMs were designed with substructures, which are contiguous, nonoverlapping subdomains of arbitrary shapes. Their main common feature is the derivation of a Schur complement on the interface between the subdomains. Overlapping subdomains techniques for parallel computing followed, e.g., as the additive Schwarz method (AS), the Lions's parallel Schwarz method, the restricted AS (RAS) and the weighted AS (WAS). Here, our interest is in the iterative primal substructuring framework, which includes, e.g., methods of the Neumann-Neumann type, the balancing DD (BDD) method, the BDD by constrained energy minimization (BDDC) method. Their dual counterparts are mostly related to the family of methods of finite element tearing and interconnecting (FETI). This is a non-exhaustive list of DDM examples. In all cases, DDMs tend to efficiently mix direct and iterative solvers, however, classical iterative methods are essentially sequential, which raises synchronization issues in distributed environments. There is therefore a growing interest in achieving asynchronous iterations.

Asynchronous iterative methods appeared by generalizing the Ostrowski's free-steering relaxation method. They were first experimented in a shared-memory environment, by removing mutual exclusion mechanisms, which let the processes freely access the shared solution vector. It resulted that there was no guarantee that the processes "see" the latest version of the whole solution vector when iterating on its components. Asynchronous DDMs early emerged with some key results including the asynchronous Lions's Schwarz method and the asynchronous WAS method. Lately, an asynchronous substructured (block) Jacobi method has appeared, which has the advantage of performing both the condensation and the iteration on the interface in one single step. A Gauss-Seidel improvement (alternation between condensation and iteration) has resulted in an asynchronous Jacobi-preconditioned Schur complement method.

To achieve DDMs that scale well with the number of subdomains, coarse spaces have been used extensively, resulting in so-called two-level iterative methods. Beside a recent implementation study of two-level asynchronous RAS methods, first mathematically modeled asynchronous coarse-space correction techniques have very recently appeared within the asynchronous WAS framework. Extension have been carried out to an asynchronous block-Jacobi-preconditioned Schur complement method, leading finally to the state-of-the-art two-level asynchronous WAS-preconditioned Schur complement method. In this presentation, we investigate a two-level asynchronous substructuring method by mostly considering practical aspects in those latest advances.

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