
Multigrid with Linear Storage Complexity

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Abstract

As the discretization error for the solution of a partial differential equation (PDE) decreases, the precision required to store the corresponding coefficients naturally increases. The storage complexity of state-of-the-art solvers therefore grows as $O(n \log n)$ where n is the number of degrees of freedom (DoFs). This talk presents a full multigrid method to compute the solution in a compressed format reducing the storage complexity of the solution and intermediate vectors to $O(n)$ bits. This allows a matrix-free implementation to solve elliptic PDEs with an overall linear space complexity. For problems limited by the memory capacity of current supercomputers, we expect a memory footprint reduction of about an order of magnitude compared to state-of-the-art mixed-precision methods. Using linear elements to solve Poisson's equation, irrespective of the problem size, as few as 4 bits for the solution and 5 bits for each of three intermediate vectors are required per DoF on the finest grid, in addition to quantities on the coarser grids and lower-order terms.

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