
On the numerical stability of the preconditioned conjugate gradient algorithm using mixed precision

Thomas Bake^{*1}, Erin Carson¹, and Yuxin Ma¹

¹Charles University – Czech Republic

Abstract

The preconditioned conjugate gradient (PCG) is very frequently method of choice for approximating the solution of sparse, symmetric positive definite linear algebraic systems. While the use of a preconditioner can significantly improve the rate of convergence, its application usually involves solving two triangular systems; a potential bottleneck in the algorithm. Employing low precision in the application of the preconditioners has the potential of overcoming this computational bottleneck, while at the same time providing a high#accuracy approximation. Multiple mathematically equivalent PCG variants have been developed, but these can exhibit a different behavior in finite precision. In particular, to the best of our knowledge, these variants have not been analyzed in the context of mixed precision under a unified framework. We provide insights into the behavior of distinct PCG variants through backward and forward stability analyses, as well as through results based on the residual gap.

^{*}Speaker